Procedure for analysis

❖ Internal loading

- Section shaft perpendicular to its axis at point where shear stress is to be determined
- Use free-body diagram and equations of equilibrium to obtain internal torque at section.

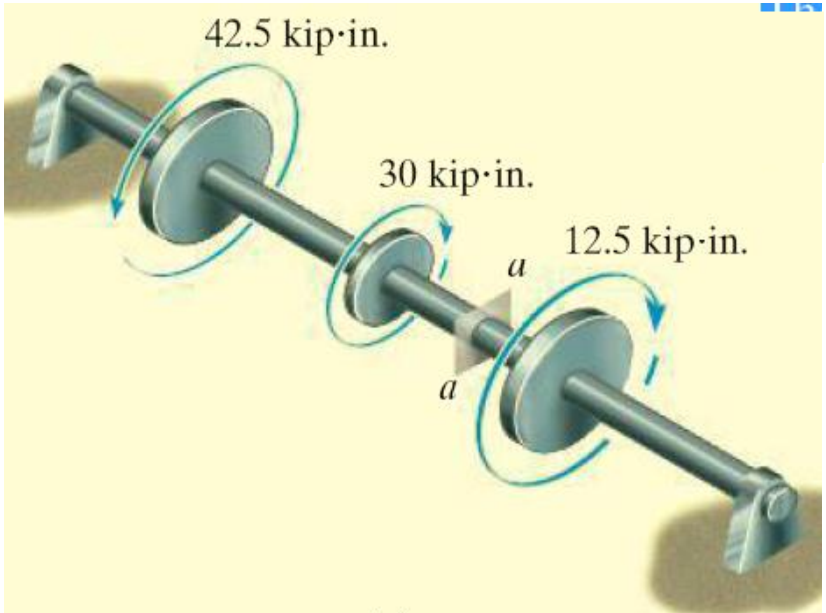
❖ Section property

- Compute polar moment of inertia and x-sectional area
- For solid section, $J = \pi c^4/2$
- For tube, $J = \pi(c_o^{\,4} - c_i^{\,2})/2$

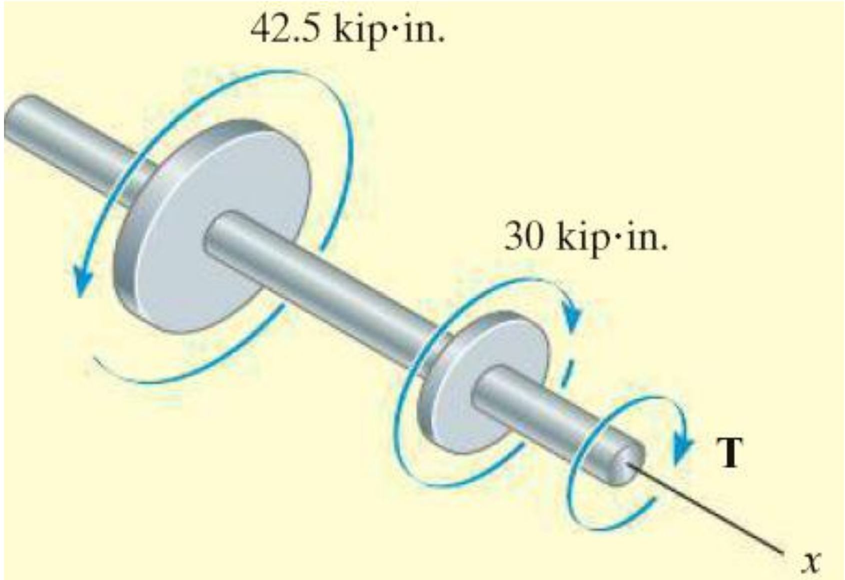
❖ Shear stress

- Specify radial distance ρ, measured from center of x-section to point where shear stress is to be found
- Apply torsion formula, $\tau = T\rho / J$ or $\tau_{\max} = Tc / J$
- Shear stress acts on x-section in direction that is always perpendicular to ρ

EX1:- The shaft shown in Fig. 5–11 *a* is supported by two bearings and is subjected to three torques. Determine the shear stress developed at points ***A*** and ***B***, located at section ***a – a*** of the shaft, Fig. 5–11 *c*.



(a)



(b)

SOLUTION

Internal Torque.

The internal torque at section *a–a* will be determined from the free-body diagram of the left segment, Fig. 5–11*b*. We have

$$\Sigma M_x = 0; \quad 42.5\text{ kip}\cdot\text{in.} - 30\text{ kip}\cdot\text{in.} - T = 0 \qquad T = 12.5\text{ kip}\cdot\text{in.}$$

Section Property. The polar moment of inertia for the sha

$$J = \frac{\pi}{2}(0.75\text{ in.})^4 = 0.497\text{ in.}^4$$

Shear Stress. Since point A is at $\rho = c = 0.75$ in.,

$$\tau_A = \frac{Tc}{J} = \frac{(12.5\text{ kip}\cdot\text{in.})(0.75\text{ in.})}{(0.497\text{ in.}^4)} = 18.9\text{ ksi}$$

Likewise for point B, at $\rho = 0.15$ in., we have

$$\tau_B = \frac{T\rho}{J} = \frac{(12.5\ \text{kip}\cdot\text{in.})(0.15\ \text{in.})}{(0.497\ \text{in.}^4)} = 3.77\ \text{ksi}$$

Directions of the stresses on elements A and B established from direction of resultant internal torque **T**.

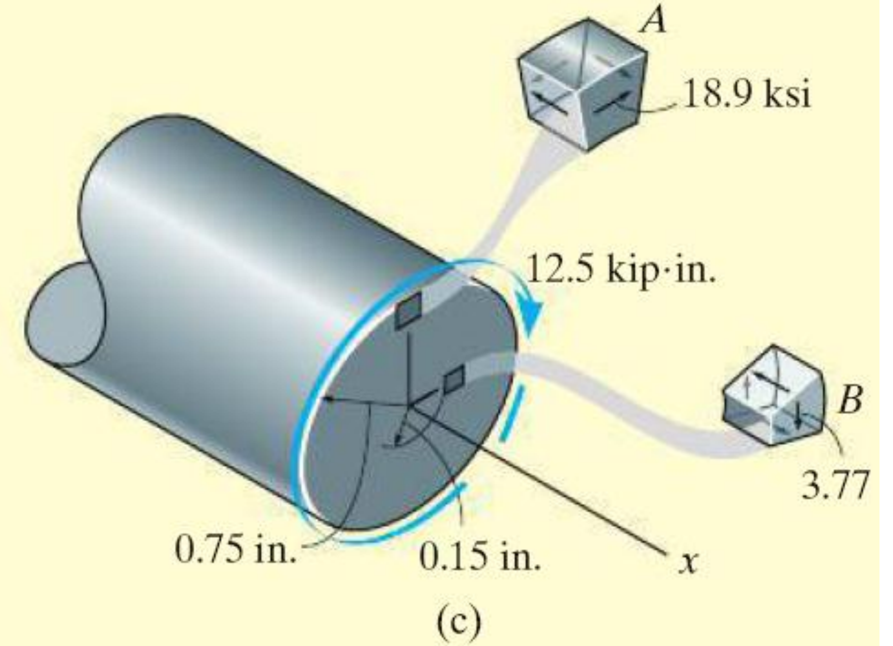
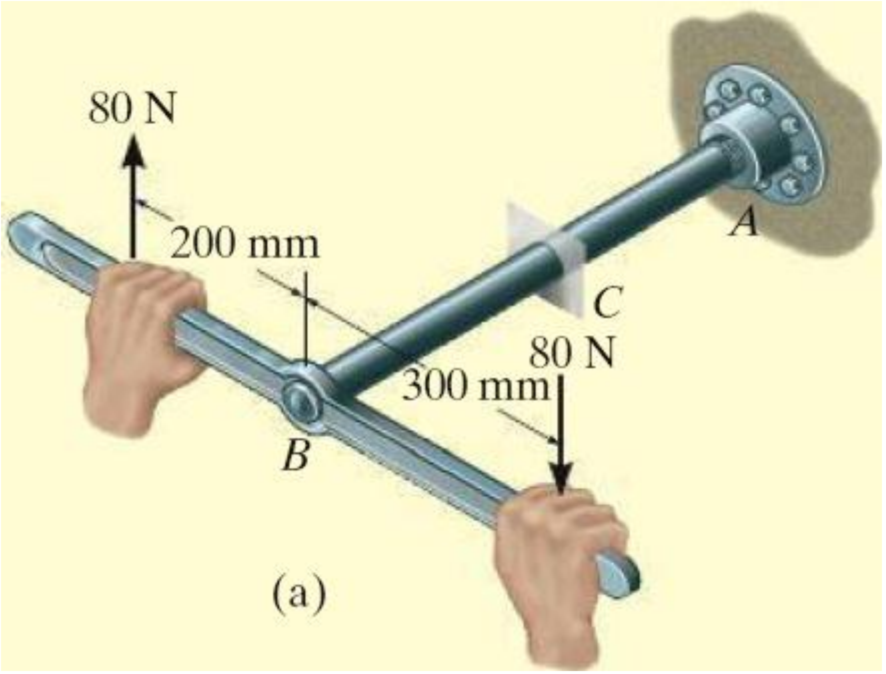


Fig. 5–11

EX2:-The pipe shown in Fig. 5–12 *a* has an inner diameter of 80 mm and an outer diameter of 100 mm. If its end is tightened against the support at *A* using a torque wrench at *B* , determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.



SOLUTION

Internal Torque. A section is taken at an intermediate location C along the pipe's axis, Fig. 5–12b. The only unknown at the section is the internal torque **T**. We require

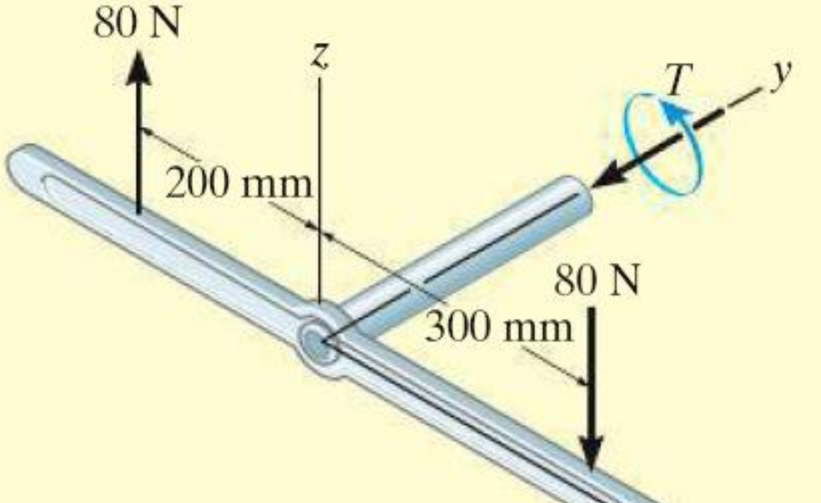
$$\Sigma M_y = 0; \quad 80\text{ N }(0.3\text{ m}) + 80\text{ N }(0.2\text{ m}) - T = 0$$

$$T = 40\text{ N}\cdot\text{m}$$

Section Property. The polar moment of inertia for the pipe's cross-sectional area is

$$J = \frac{\pi}{2}\left[(0.05\text{ m})^4 - (0.04\text{ m})^4\right]$$

$$= 5.796(10^{-6})\text{ m}^4$$



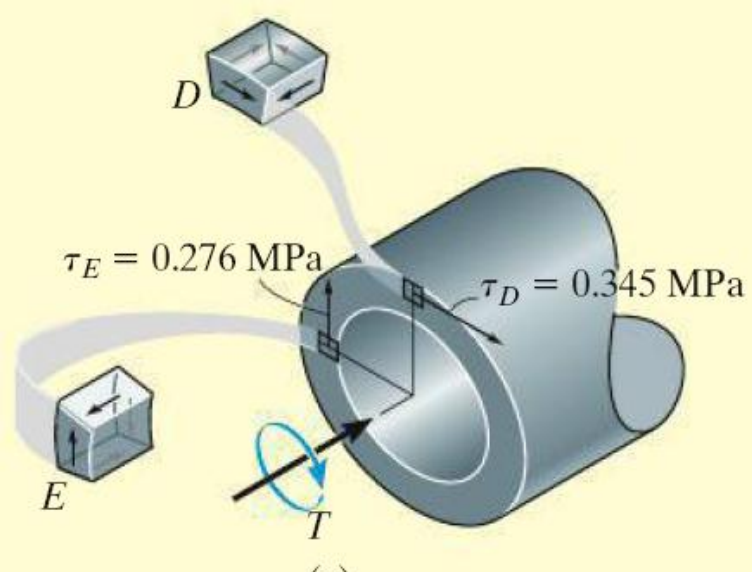
(b)

Shear Stress. For any point lying on the outside surface of the pipe, $\rho = c_o = 0.05$ m, we have

$$\tau_o = \frac{Tc_o}{J} = \frac{40 \text{ N}\cdot\text{m}(0.05 \text{ m})}{5.796(10^{-6})\text{m}^4} = 0.345 \text{ MPa} \qquad \textit{Ans.}$$

And for any point located on the inside surface, $\rho = c_i = 0.04$ m, so that

$$\tau_i = \frac{Tc_i}{J} = \frac{40 \text{ N}\cdot\text{m}(0.04 \text{ m})}{5.796(10^{-6})\text{m}^4} = 0.276 \text{ MPa} \qquad \textit{Ans.}$$



(c)

5.3 POWER TRANSMISSION

- Shafts and tubes having circular cross sections are often used to transmit power developed by a machine.
- When used for this purpose, they are subjected to a torque that depends on the power generated by the machine and the angular speed of the shaft.

The belt drive transmits the torque developed by an electric motor to the shaft at A. The stress developed in the shaft depends upon the power transmitted by the motor and the rate of rotation of the shaft. $\boldsymbol{P = T\omega}$.

- ***Power*** is defined as the work performed per unit of time.

- The work transmitted by a rotating shaft equals the torque applied times the angle of rotation.

- If during an instant of time $\boldsymbol{dt}$ an applied torque $\boldsymbol{T}$ causes the shaft to rotate $\boldsymbol{d\theta}$, then the instantaneous power is

Units

In the SI system, power is expressed in *watts* when torque is measured in newton-meters (N.m) and $\boldsymbol{\omega}$ is in radians per second (rad/s) (1 W = 1 N. m/s).

In the FPS system, the basic units of power are foot-pounds per second (ft. lb/s); however, horsepower (hp) is often used in engineering practice where

$$1 \text{ hp} = 550 \text{ ft} \cdot \text{lb/s}$$

For machinery, the *frequency* of a shaft's rotation, f, is often reported. This is a measure of the number of revolutions or cycles the shaft makes per second and is expressed in hertz (1 Hz = 1 cycle/s). Since 1 cycle = 2π rad, then $\omega = 2\pi f$, and so the above equation for power becomes

$$\boxed{P = 2\pi f T} \tag{5–11}$$

Shaft Design

- If power transmitted by shaft and its frequency of rotation is known, torque is determined from Eqe 5-11.

$$T = P/2\pi f.$$

- Knowing T and allowable shear stress for material, τ_{allow} and applying torsion formula,

$$\frac{J}{c} = \frac{T}{\tau_{\text{allow}}}$$

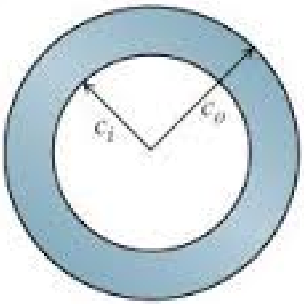
Strength of Materials: Second Class

- For solid shaft,

substitute $J = (\pi/2)c^4$ to determine c

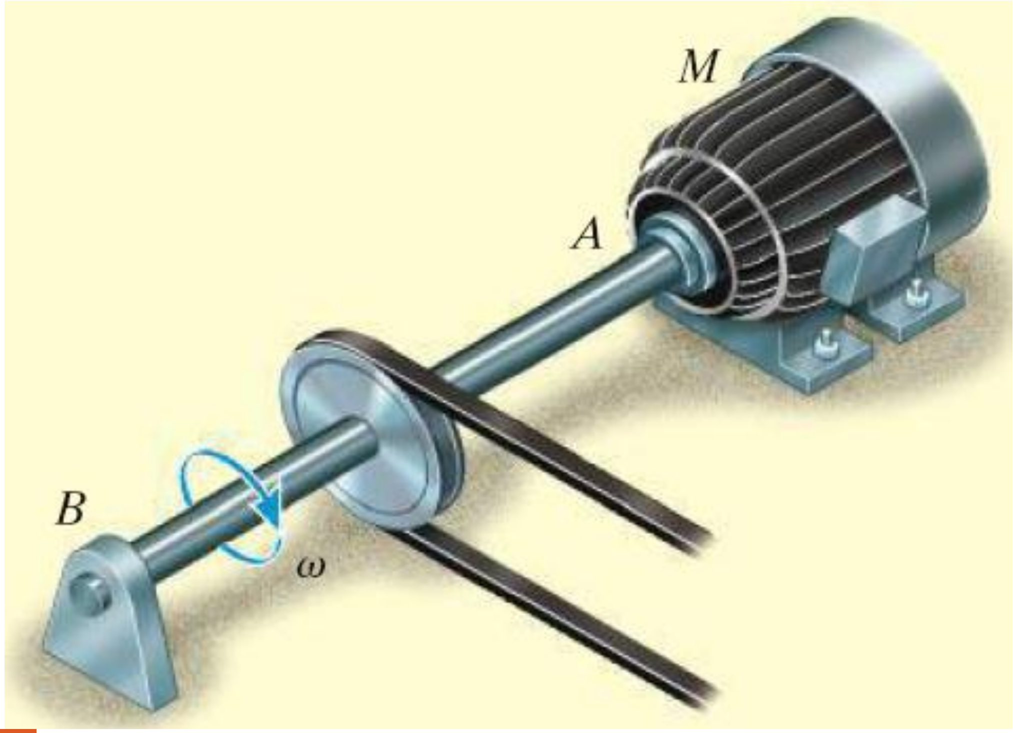
- For tubular shaft,

substitute $J = (\pi/2)(c_o^4 - c_i^4)$ to determine c_o and c_i



Ex

A solid steel shaft AB, shown in Fig. 5–13, is to be used to transmit 5 hp from the motor M to which it is attached. If the shaft rotates at $\omega = 175$ rpm and the steel has an allowable shear stress of $\tau_{allow} = 14.5$ ksi, determine the required diameter of the shaft to the nearest $\frac{1}{8}$ in.



SOLUTION

The torque on the shaft is determined from Eq. 5–10, that is, $P = T\omega$. Expressing P in foot-pounds per second and ω in radians/second, we have

$$P = 5\text{ hp}\left(\frac{550\text{ ft}\cdot\text{lb/s}}{1\text{ hp}}\right) = 2750\text{ ft}\cdot\text{lb/s}$$

$$\omega = \frac{175\text{ rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 18.33\text{ rad/s}$$

Thus,

$P = T\omega;$

$$2750\text{ ft}\cdot\text{lb/s} = T(18.33\text{ rad/s})$$

$$T = 150.1\text{ ft}\cdot\text{lb}$$

Applying Eq. 5–12 yields

$$\frac{J}{c} = \frac{\pi}{2}\frac{c^4}{c} = \frac{T}{\tau_{\text{allow}}}$$

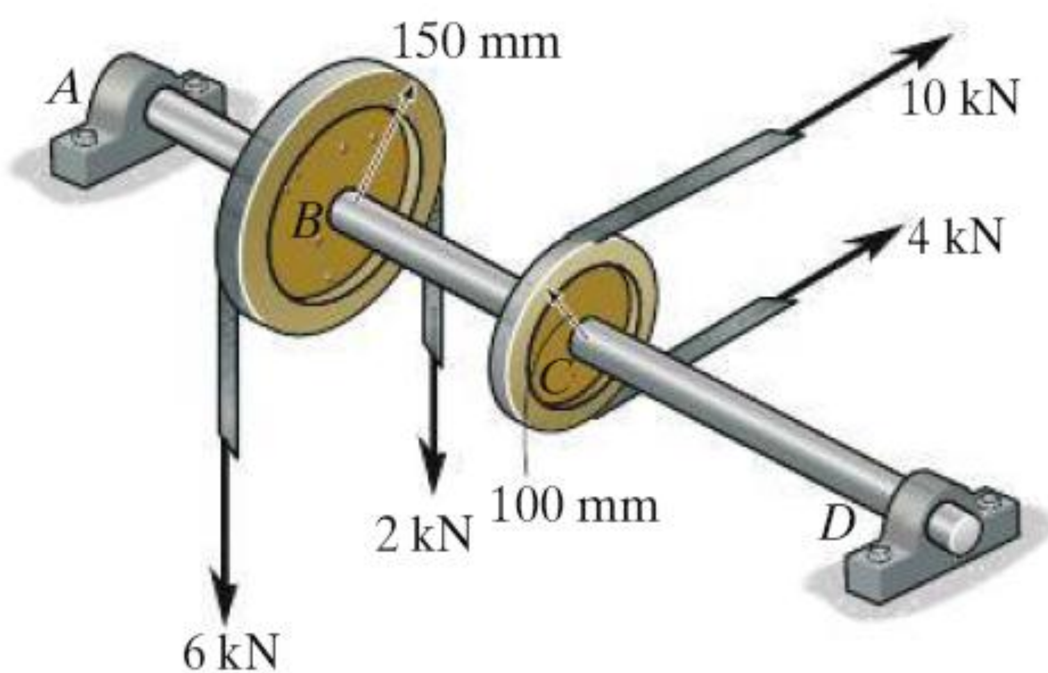
$$c = \left(\frac{2T}{\pi \tau_{\text{allow}}}\right)^{1/3} = \left(\frac{2(150.1\ \text{ft} \cdot \text{lb})(12\ \text{in./ft})}{\pi(14\ 500\ \text{lb/in}^2)}\right)^{1/3}$$

$$c = 0.429\ \text{in.}$$

Since $2c = 0.858$ in., select a shaft having a diameter of

$$d = \frac{7}{8}\ \text{in.} = 0.875\ \text{in.} \qquad \textit{Ans.}$$

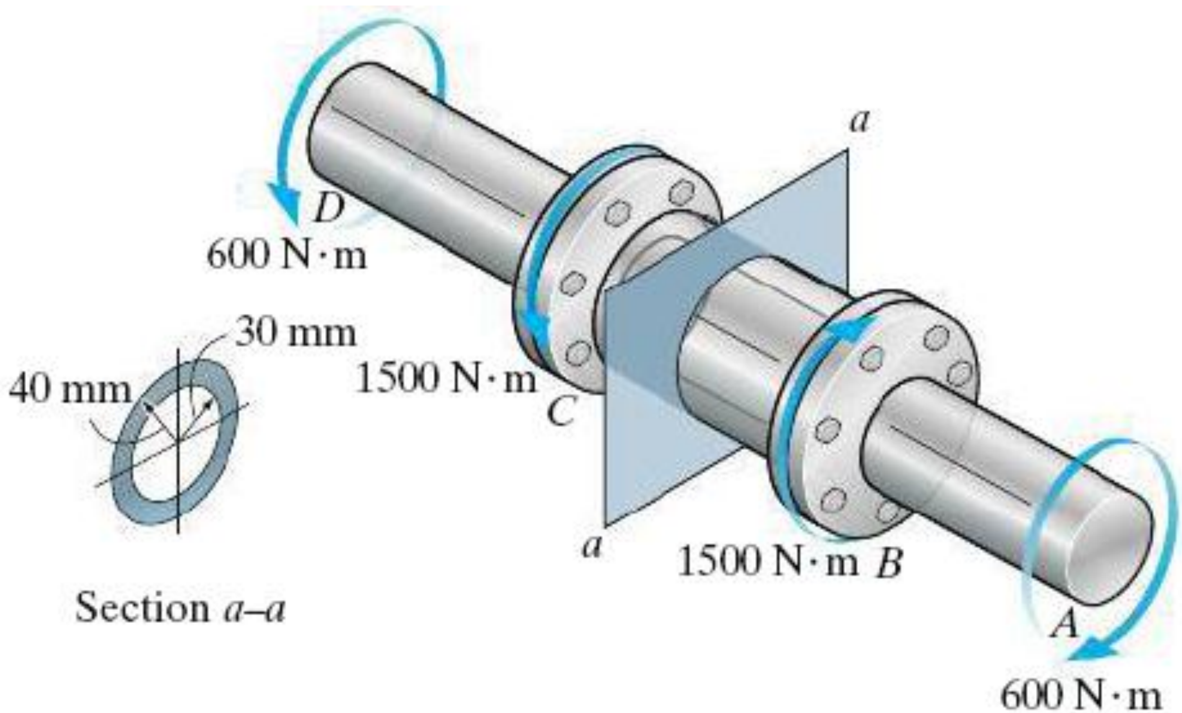
F5–4. Determine the maximum shear stress developed in the 40-mm-diameter shaft.



F5–4

TORSION

F5–5. Determine the maximum shear stress developed in the shaft at section *a*–*a*.



F5–5